at a set of related frequencies whose ratios are specified in (2). Data of the sort desired may already be available from some laboratory engaged in ionospheric soundings. If not, it is suggested that simultaneous soundings at several frequencies be taken and the results reported. If the experiments gave a positive result, changes obviously would be necessitated in some of our accepted views as to the structure of the ionosphere.

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Stability Criteria for Tunnel-Diode Amplifiers*

A tunnel-diode amplifier is stable when the amplifier network is reduced to an arbitrary single loop and the equation given by the sum of the impedances around this loop equal to zero [$\Sigma Z(P)=0$ where $p=\gamma+j\omega$] has no solution in the right-half plane ($\gamma>0$). This is equivalent to the requirement that the system determinant shall have no zeros in the right-half plane. Several authors have used this criterion.[1,2] To determine analytically whether $\Sigma Z(p)$ has any positive zeros is very laborious if at all possible for many practical amplifier configurations.

A possible approach is to use a contour theorem from the theory of complex functions as formulated by Goldman.[3] "If a function $Z(p)$ is analytic, except for possible poles within a given contour taken in the clockwise direction in the p-plane, then the number of times that this contour into the Z-plane encircles the origin in the Z-plane in a clockwise direction is equal to the number of zeros minus the number of poles of $Z(p)$ inside the contour in the p-plane, each pole and zero being counted in accordance with its multiplicity."

When applying this contour theorem to $\Sigma Z(p)$ for an arbitrary loop in the tunnel diode circuitry, we must have a complete knowledge of the poles of $\Sigma Z(p)$ in the right-half plane and at infinity. Hughes[4] discusses these difficulties and obtains a plot easy to interpret by dividing the original function $Z(p)$ with a new function $Z_2(p)$ with no zeros in the right-half p-plane, the same number of poles as $Z(p)$ in the right-half p-plane and with the same limit at infinity as $Z(p)$ This method is applicable to tunnel-diode amplifiers, but a complete knowledge of the poles in the right-half p-plane is sufficient to interpret the complex plot of $\Sigma Z(p)$ along a chosen contour. Generally we can say that the closer to the negative resistance we choose the loop representing the tunnel diode and amplifier network, the fewer the poles in the right-half p-plane. It is convenient to choose the representation given in Fig. 1 for then the diode is separated from the rest of the network.

$Z'(p)$ includes the cartridge capacitance, bias circuit, matching network, load, etc., and is a passive impedance. The impedance around this loop is given by

$$\sum Z(p) = \frac{\frac{1}{C}}{p - \frac{1}{RC}} + R_s + pL + Z'(p).$$

This function has one pole in the right-half plane at $p=\gamma_p=1/RC$ and a pole at $p=\pm\infty$.

With a contour in the p-plane chosen as the $j\omega$-axis and a semicircle enclosing all poles and zeros in the right-half plane, the ΣZ-plot of a stable amplifier encircles the origin once in a counterclockwise direction as determined by the pole $p=1/RC$. From symmetry it is sufficient to plot $\Sigma Z(j\omega)$ for positive frequencies and only up to the diode cutoff frequency ω_c as the circuit is passive above ω_c. Davidsohn, Hwang and Ober[5] have considered stability criteria from a similar point of view.

Summing up we have the following stability criterion: "*A tunnel-diode amplifier is stable if and only if the sum of the diode impedance and the connected network impedance plotted as a function of frequency encircles the origin once in a counterclockwise direction when the plot is closed with an arbitrary line in the right-half Z-plane between the positive and negative diode cutoff frequency. The diode cartridge capacitance is considered to belong to the network connected to the diode.*"

Fig. 2 shows the application of this criterion. This circuit is one of the simplest possible amplifier circuits and yet it is laborious to investigate the stability analytically.

The graphical display gives a good feeling of how the amplifier stability is influenced by change in diode parameters. This is especially true when the amplifier circuit is more complex than given in the example so that the minimum distance between the origin and the plotted function occurs for frequencies different from the amplifier center frequency.

When an amplifier configuration connected to an ideal transmission line or a load resistance R_0 is determined to be stable, it is of interest to determine which mismatch is permissible at the input without upsetting the stability. To do this we reduce the amplifier network to the loop nearest to the transmission line connection (Fig. 3). Z_{ampl} includes the tunnel diode and matching network Z_{load} is the transmission line impedance with mismatches from circulators, stabilizing networks, etc. A condition for stability is that $Z_{ampl}(p)+Z_{load}(p)=0$ has no solutions in the right-half plane. If we define

$$G = \text{(ideal voltage gain)} = \frac{Z_{ampl} - R_0}{Z_{ampl} + R_0}$$

$$\rho = \text{(nonideal load reflection coefficient)} = \frac{Z_{load} - R_0}{Z_{load} + R_0},$$

then $Z_{ampl}+Z_{load}=0$ can be rewritten as $G\cdot\rho-1=0$.

As $G\cdot\rho$ has no poles in the right-half plane (G is stable and Z_{load} is a passive impedance), a necessary and sufficient stability criterion is that the complex plot of $G\cdot\rho$ when ω goes from $-\infty$ to $+\infty$ does not encircle the point $+1$. (Compare with the Nyquist criterion for feedback-amplifiers.)

If the phase of the input reflection coefficient is not known or not controlled, a sufficient criterion for stability is

$$|G|\cdot|\rho| < 1.$$

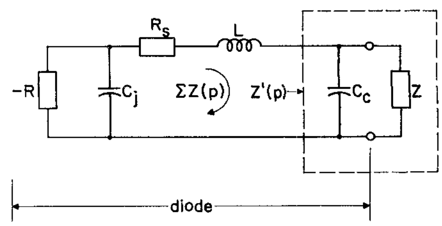

Fig. 1—Equivalent tunnel-diode amplifier representation showing loop chosen for stability criteria.

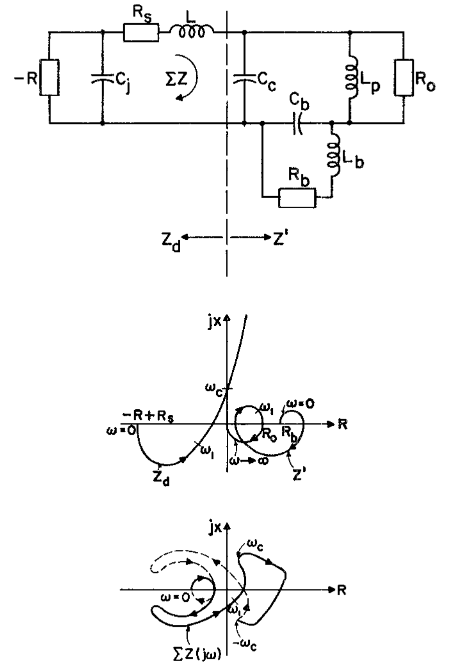

Fig. 2—Example of diode and connected network representating a stable amplifier.

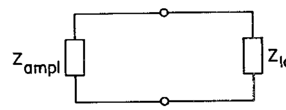

Fig. 3—Loop for determining permissible mismatch.

* Received March 26, 1962; revised manuscript received, May 4, 1962.

[1] H. Boyet, D. Fleri and C. A. Renson, "Stability criteria for a tunnel diode amplifier," PROC. IRE, vol. 49, p. 1937; December, 1961.

[2] L. L. Smilen and D. C. Youla, "Stability criteria for tunnel diodes," PROC. IRE, vol. 49, pp. 1206–1207; July, 1961.

[3] S. Goldman, "Transformation Caculus and Electrical Transients," Prentice-Hall, Inc., New York, N. Y., pp. 370–371; 1950.

[4] W. L. Hughes, "Nonlinear Electrical Networks," Ronald Press Co., New York, N. Y.; pp. 166–168, 1960.

[5] U. S. Davidsohn, Y. C. Hwang and G. B. Ober, "Designing with tunnel diodes, part 1," *Elec. Design*, vol. 8, pp. 50–55; February, 3, 1960.

It is considered that a combined use of the above described criteria gives a practical means to investigate the stability of complex tunnel-diode circuits and to determine the limits of permissible mismatch and diode characteristics variation.

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On "An Impedance Transformation Method for Finding the Load Impedance of a Two-Port Network"*

The above article[1] begins with ten extensive footnotes, but the authors omit the one reference that discusses their problem: Deschamps' "Hyperbolic Protractor."[2] The undersigned writers concede that the method of Mittra and King is different, but it is more complicated and less useful than that of Deschamps.

Concerning footnote 15, one notes that $R_{11}R_{22}-R_{12}^2 \geq 0$ is only a necessary but not sufficient condition for a positive definite quadratic form, and $R_{11}>0$, or $R_{22}>0$ is also required for sufficiency.

The undersigned find it somewhat surprising that many of the techniques found in Deschamps' pamphlet are not more widely used, for they apply to the interesting problems in measurements on linear passive reciprocal two-ports.

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* Received March 1, 1962.

[1] R. Mittra and R. J. King, IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 13–19; January, 1962.

[2] G. A. Deschamps, "A Hyperbolic Protractor for Microwave Impedance Measurements and Other Purposes," Federal Telecommunication Labs., Nutley, N.J.; 1953. [See especially problems 7 (lossless case) and 13 (lossy case).]

Authors' Comment[3]

Concerning our paper,[1] Messrs. Stock and Kaplan have made the comment that our method for calculating an unknown load through a junction is complicated. To illustrate their point, they cite two examples (7 and 13) given in a booklet by Deschamps.[2,4]

We feel that Stock and Kaplan have missed the most important points in our paper. Section II of our paper is devoted to the establishment of a linear relationship between the input reflectance Γ_{in} and a modified load reflectance Γ_L' by $\Gamma_{in}-\bar{a}=\bar{b}(1-\Gamma_L')$. The constants $\bar{a}$ and $\bar{b}$ may be easily found but are not necessary for calibration. This linear relationship is the very heart of our paper, just as Deschamps' invariance of "hyperbolic" distances and "elliptic" angles is the basis of his booklet.

The examples chosen for comparison by Stock and Kaplan involve the so-called "three point method" which is subject to experimental errors. In any case, it is instructive to compare the two methods of solution of a typical example and let the reader decide which is more complicated and which is more accurate.

Example (*Problem 5—Lossless Case*): We have chosen problem 5 instead of 7 which is essentially the same but has additional property of more nearly showing the inverse transformation from load to input as well as the transformation from Γ_{in} to Γ_L'. The choice of the input reference is arbitrary so let us rotate the data given by Deschamps[2] clockwise 71° on the Smith chart so that when $\Gamma_L=+1 (Z_L=\infty)$, $\Gamma_{in}=+1$. The data would then read as follows:

1) $Z_{in}=j90$ when $Z_L=0$,
2) $Z_{in}=\infty$ when $Z_L=\infty$,
3) $Z_{in}=81+j90$ when $Z_L=Z_{02}=200\ \Omega$.

What is the input impedance for a termination of $Z_L=520\ \Omega$?

Take $Z_{01}=100\ \Omega$ as the center of the input reflectance chart. It should be pointed out the original data given in the booklet has some error in that the input impedance corresponding to $Z_L=0$ should be $j12$ rather than $j10$ in order for the other two measurements to be consistent. This error may be fairly difficult to detect with the hyperbolic protractor because of the relatively large hyperbolic distances (measured in db) involved in this example.

Solution (*Mittra and King*): We have chosen the input reference such that the Γ_{in} plane coincides with the Γ_L' plane. Hence, using the transformation $z_L' \equiv r_L'+jx_L' = r_L/r_1+j(x_L+x_1)/r_1$ and data 3) above, $r_1=1/0.80=1.233$ and $x_1/r_1=0.90$ for $z_L=1$, which determines the calibration constants r_1 and x_1. To obtain $z_L'=z_{in}$ for $Z_L=520\ \Omega$ we again apply the transformation relating z_L' to z_L. ($r_L=2.60$, $x_L=0$.) $z_L'=z_{in}=r_L/r_1+jx_1/r_1=2.105+j0.90$ which completes the discussion. It is obvious that the inverse problem, *i.e.*, that of finding Z_L when Z_{in} is given is just reciprocal.

Solution (*Hyperbolic Protractor Method*): Since many readers do not have access to the booklet describing the use of the hyperbolic protractor we reproduce the solution in Fig. 1. Plot Q', P' and O' corresponding to data 1), 2) and 3) above. When the output port is matched (200 Ω), the corresponding input reflectance point is O' which is called the "iconocenter" by Deschamps. The transformation may be made to the projective chart by constructing $\bar{O}=\beta(O')$. Distances on the reflectance (Smith) charts are denoted by [ ] and by ⟨ ⟩ on the projective charts. Reflectances at the input port are denoted by primes and points on the projective chart by bars. Thus $\langle O\bar{O}\rangle = 2[O\bar{O}']$ = 17 db on Fig. 1. The points P' and Q' do not change in this transformation β and therefore the point $\bar{O}$ should fall on the straight line $\overline{QP}$, the image of the diameter QP. The point $\bar{W}$ which represents the input reflectance for 520 Ω at the output will be on $\overline{QP}$ at the hyperbolic distance $\langle \overline{OW}\rangle = [OW] = 8$ db or 16 db as measured on the projective chart. This immediately gives a means for constructing $\bar{W}$ which should be between $\bar{O}$ and P' since W itself lies between O and P. Measuring $\langle O\bar{W}\rangle$ with the protractor it is found to be 16 db, and W' is obtained by taking the hyperbolic midpoint of $\langle O\bar{W}\rangle$ or 8 db. The corresponding impedance obtained from the reflectance chart is then $2.06+j0.90$.

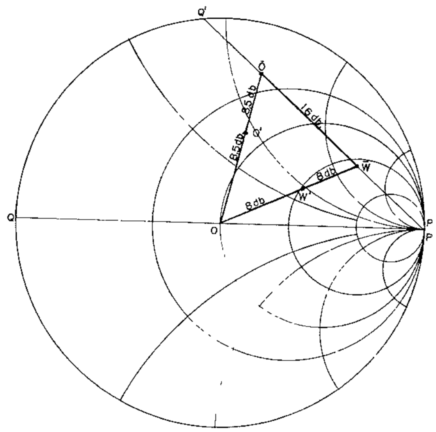

Fig 1.

Example (*Problem 13—Lossy Case*): The given data are:

1) Iconocenter $=8$ db$/-90°$,
2) $\Gamma_{in}=9$ db$/28°$ when $Z_L=0$.

What is the unknown load impedance when $\Gamma_{in}=10$ db$/-134°$?

Solution (*Mittra and King*): This example shows how one deals with the image circle rather than the unit circle but follows the same steps.

Expand the Γ_{in}-circle linearly and rotate it to correspond to the Γ_L'-circle. Read the transformed iconocenter from the Γ_L' plane as $z_{LO}'=0.625-j0.65=r_L/r_1+j(x_L+x_1)/r_1$. The load corresponding to this point is $z_L=1+j0$, so $r_1=1.60$ and $x_1=-1.04$. Now read the transformed point corresponding to the unknown load impedance as $z_L'=0.375-j0.043$. Using the impedance transformation equation as before we find $z_L=0.60+j0.991$. Thus, the actual load reflectance is $\Gamma_L=11.1$ db$/80.5°$. Incidentally, Deschamps' angle $(\overline{CP''}, \overline{CL''})=80.5°$, not 71°, in problem 13.

[3] Received June 1, 1962.

[4] G. A. Deschamps, "A new chart for the solution of transmission line and polarization problems," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-1, pp. 5–13; March, 1953. This paper describes the theory discussed in Deschamps "Hyperbolic Protractor" and is included in that booklet.